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Double Best Response Dynamics in Topology Formation Game for Ad Hoc Networks

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Abstract—This paper considers a topology formation problem for wireless ad hoc networks. There are wireless nodes located on a plane. Each node can adjust its transmission power in the dynamic mode. The global objective lies in assigning an optimal transmission power to each node so that the resulting topology is connected and minimizes the total power cost. The topology formation problem is studied as a noncooperative game. The author proposes two algorithms of collective behavior and network formation based on the so-called "double best response" decision rule. This decision rule originates from the reflexive game framework and describes the behavior of an agent with reflexion rank 1. The efficiency of the suggested algorithms is evaluated by simulation and compared with the standard best response algorithm.

1. INTRODUCTION

Ad hoc networks are constructed from several wireless nodes without any auxiliary infrastructure [23]. There exist different types of ad hoc networks. Mobile ad hoc networks (MANET) are applied in military and rescue operations. Sensor networks (WSN) serve for data acquisition in industrial objects, ecological monitoring in cities and nature exploration. Nowadays, researchers study the feasibility of involving medium-priced ad hoc networks for increasing the capacity and extending the coverage area of 3G and 4G mobile networks, see [10].

Generally, devices in ad hoc networks are powered by autonomous accumulators. Therefore, a major role belongs to energy efficiency methods. Topology control means dynamic adjustment of transmitter power for maintaining network connectivity, minimizing power consumption or improving communication quality. We consider the process of network formation only: for each node, it is necessary to assign a transmission power so that the resulting network is connected and minimizes the total power cost.

According to the classical interpretation of game theory, agents are subjects possessing freedom of will but preferring rational actions (i.e., maximizing their utility function). Originally, game theory studied situations, where agents represent commercial firms, political parties and any other entities with decision-making by human beings. At the current stage of development, game theory methods appear applicable to technical systems, particularly, telecommunication systems.

Devices in technical systems are unable to make decisions. However, a designer can define a behavioral algorithm for a device, which imitates rational decision-making. Suppose that each device has an embedded utility function assessing its performance (tasks' fulfilment) depending on the objectives and actions of the device and the state of an external environment. A system includes several such devices and each of them strives for maximizing its individual utility function without due account of other utility functions. The stated class of problems is analyzed by theory of noncooperative games and devices are accordingly treated as agents. Standard control and optimization methods can be employed instead of game theory under an existing coordination algorithm for devices.

This paper focuses on a decision rule of agents called double best response. The network topology formation game under consideration is remarkable for the presence of very many equilibria differing from each other in efficiency. Equilibria yielded by the classical game-theoretic algorithm of iterated best response strongly depend on the sequence of agents' actions. We establish the following fact. In the case of network formation by the double best response algorithm, the result enjoys better stability with respect to the sequence of actions and guarantees smaller mean total cost.

Furthermore, two network formation algorithms are introduced. In the first algorithm, network nodes involve double best response until the network gets stabilized, and then switch to standard best response to complete connected network formation. According to the second algorithm, nodes use simple best response while improving their utility and then switch to double best response. Here a node can apply double best response only a bounded number of times. In comparison with standard best response, both algorithms have demonstrated higher network performance in numerical experiments.

The paper is organized as follows. Section 2 briefly surveys the publications dedicated to game theory application in wireless network control problems. Next, Section 3 formulates the topology control game. The double best response rule and its modifications are stated in Section 4. The suggested network formation algorithms are described in Section 5. And finally, Section 6 presents the results of numerical experiments, whereas the conclusions and directions of further investigations are discussed in Section 7.

2. OVERVIEW

As a matter of fact, numerous works are dedicated to topology control in wireless networks. They differ in the models of networks, performance criteria and constraints imposed on networks. For a detailed discussion of theoretical results in this field, we refer to [21]. Searching for a solution minimizing the total power cost of nodes is an *NP*-complex problem [9]. Centralized algorithms of integer optimization and some heuristics were provided in [8].

In addition, many decentralized algorithms of topology control were developed. For instance, the LMST algorithm [16] performs decentralized search of minimum spanning tree. For this, nodes have to possess information on the location of their neighbors. The CBTC algorithm requires that nodes can define the direction to other nodes [17]. In the XTC algorithm, each node ranks its neighbors by signal quality and then creates k best connections [25]. Modern algorithms with higher practical applicability were considered in [7].

Over the recent years, game theory methods have been involved to model conflict situations arising in telecommunication networks, particularly, wireless networks. The paper [22] gave a comprehensive overview of game theory usage in sensor networks; many problems described there are characteristic for other types of networks. In [11, 15] topology control was stated as a noncooperative game with a potential function, whose solution makes a Nash equilibrium. The publication [24] explored a Bayesian game of different-type nodes, where solution represents a Bayesian Nash equilibrium. In [18], nodes used learning dynamics of stochastic fictious play in a repetitive game (similarly, its solution is a Nash equilibrium).

The key achievements of theory of reflexive games were described in the book [5]. A collective behavior algorithm resembling double best response in many aspects was adopted in [4] for controlling a group of agents penetrating through a system of sensors. Using the framework of reflexive games, the monograph [3] studied the Cournot duopoly, consensus search in a multi-agent system and active expertise. Double best response was initially applied to the network formation problem in [1].

Topology control has much in common with formation games in socioeconomic networks [2]. They examine different concepts of stable networks, as well as dynamic network formation procedures [14]. However, as far as the author knows, there exist no research works applying theory of reflexive games to network formation problems.

3. WIRELESS NETWORK FORMATION GAME

3.1. Network Model

A network consists of a set of devices or nodes, $N = \{1, \ldots, n\}$, located on a plane. Each device is equipped with a wireless transmitter having a variable power. The powers of all transmitters are defined by the vector $p = (p_1, \ldots, p_n)$, $p_i \in [0, p^{max}]$. We believe that nodes appear homogeneous and possess an identical maximum transmission power.

Node i can successfully transmit data to node j under the condition

$$\frac{p_i}{(\delta_{ij})^\alpha} \geq \beta, \tag{1}$$

where p_i denotes the transmission power of node i, $\beta \geq 1$ is a parameter specifying the required quality of transmission, $\alpha \geq 2$ indicates the signal attenuation rate and δ_{ij} corresponds to the Euclidean distance between nodes i and j. This model describes radio signal propagation in a homogeneous medium without external disturbances, re-reflection and other real-condition phenomena. Nevertheless, such model is commonly encountered in topology control [21].

Definition 1. A set of nodes satisfying the condition (1) if node i adjusts power p_i is called the outgoing p_i-neighborhood of node i and designated by $N_i^{out}(p_i)$.

Definition 2. A communication graph generated by a power vector p is a directed graph $g(p) = (N, E(p))$, where N represents the set of vertices corresponding to network nodes, $E(p)$ means the set of edges, where $(i, j) \in E(p)$ if nodes i and j meet the condition (1).

Ad hoc networks operate on wireless communication protocols from Standards 802.11 [12] and 802.15 [13] with acknowledgement messages ("delivery reports"). This mechanism calls for two-way communication between nodes i and j. Therefore, we introduce another definition.

Definition 3. A connected undirected subgraph of a graph $g = (N, E)$ is a graph $\bar{g} = (N, \overline{E})$, where $\overline{(i, j)} \in \overline{E}$ if $(i, j) \in E$ and $(j, i) \in E$.

Let p^{max} be the power vector, where each node adjusts its maximum power. Accordingly, denote by $g^{max} = g(p^{max})$ the graph generated by this vector.

The performance criterion of a network is the total power of all nodes:

$$C^{total}(p) = \sum_{i \in N} p_i. \tag{2}$$

During topology formation, one has to assign powers p to nodes so that $g(p)$ comprises a connected undirected subgraph and the total power (2) of all nodes gets minimized.

3.2. Game Description

Here we address the topology formation game stated in [15].

Definition 4. A strategic-form game is the triplet

$$\Gamma = \langle N, \{A_i\}_{i \in N}, \{u_i\}_{i \in N} \rangle, \tag{3}$$

where $N = \{1, \ldots, n\}$ gives the set of players (agents); A_i makes the admissible action set of agent i; $u_i\colon \times_{i \in N} A_i \to \mathbb{R}$ are the utility functions of agents.

In the topology formation game, the role of agents belongs to network nodes $N = 1, \ldots, n$. The action of agent i is the choice of its transmission power $p_i \in [0, p^{max}]$. The utility function reflects the requirements of network connectivity and reduced transmission power:

$$u_i(p) = Mf_i(g(p)) - p_i. \tag{4}$$

For node i, the value $f_i(g(p))$ specifies the number of nodes it is connected to in the graph $g(p)$. Here we take into account nodes with existing paths from bidirectional edges. The constant $M > p^{max}$ characterizes the priority of network connectivity over power cost minimization. If necessary, the function (4) can be modified for proper consideration of other criteria (e.g., the length of paths in the graph $g(p)$).

3.3. Equilibria in Topology Formation Game

Definition 5. An action profile a^* is termed a Nash equilibrium if for each agent i and any action $a_i \neq a_i^*$ we have the condition

$$u_i(a_i, a_{-i}^*) \leq u_i(a_i^*, a_{-i}^*). \tag{5}$$

Imagine that a Nash equilibrium has been reached in the system. Then none of agents can increase its utility by unilateral deviation of its action. The topology formation game in question admits several Nash equilibria. The problem is that they may differ dramatically in the sense of the total power cost.

Figure 1 demonstrates all admissible equilibria arising in the network formation game for three nodes located on the segment of length 2. Node b is shifted by a small quantity ε with respect to the middle of the segment.

Interestingly, the equilibria in Figs. 1a and 1b do not represent the admissible solutions of the network formation problem, as the corresponding graph turns out disconnected. The equilibrium in Fig. 1c is admissible but nonoptimal. Only the equilibrium in Fig. 1d enjoys optimality in the sense of total power cost minimization. Thus, we are concerned with algorithms eliminating inadmissible and suboptimal equilibria.

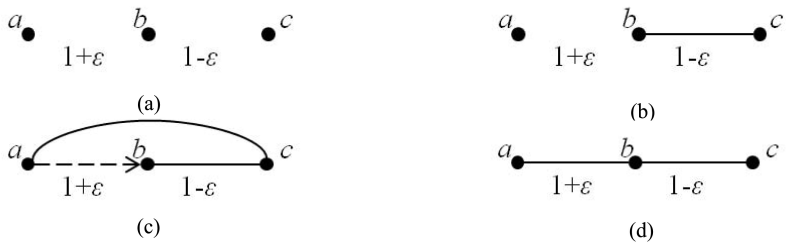


Fig. 1. Equilibria in the network formation game for three nodes: (a) trivial inadmissible equilibrium; (b) another inadmissible equilibrium; (c) suboptimal equilibrium; (d) optimal equilibrium.

4. COLLECTIVE BEHAVIOR OF AGENTS

4.1. Best Response

Analysis of the strategic-form game makes it clear which equilibria exist in this game. The choice mechanism for one of these equilibria is defined by the collective behavior algorithm of agents. Actually, this is an iterated process, where at each step agents choose their actions by some local rule. In the foreign scientific literature, the described mechanism is also called a learning process. In fact, the iterated best response algorithm makes a most widespread and natural equilibrium search algorithm.

Definition 6. The best response of agent i to an opponents' action profile a_{-i} is the action

$$BR_i(a_{-i}) = \arg\max_{x \in A_i} u_i(x, a_{-i}). \tag{6}$$

The iterated best response algorithm is defined as follows. Fix an initial action vector a^0. Agents act by turn according to some sequence. Generally, the sequence of their moves is random. At step k, agent i chooses its action as the best response to the current opponents' action profile a_{-i}^{k-1}.

The iterated best response algorithm depends on the initial state a^0 and on the sequence of nodes' actions. For instance, the equilibrium in Fig. 1c takes place if nodes begin with the maximum transmission power and node b varies its power first. On the other hand, if first actions belong to node a or c, we arrive at the equilibrium illustrated by Fig. 1d. In the case when the initial power of all nodes is insufficient for forming at least one connection, the trivial equilibrium occurs under any sequence of actions, see Fig. 1a.

In comparison with best response, the following algorithm has better stability with respect to initial actions and the sequence of moves.

4.2. Double Best Response

The best response (6) models the behavior of a "short-sighted" agent who believes in the invariability of the opponents' action profile. Theory of reflexive games developed in [5] considers agents trying to predict the future response of the opponents to their actions.

The book [5] introduces the notion of agent's reflexion rank. Agents having reflexion rank 0 adhere to the simple best response (6). Next, an agent with reflexion rank 1 considers that all other agents have reflexion rank 0. We will not analyze reflexion ranks higher than 1. Their usage in game theory applications is described in the monograph [3].

Agents with reflexion rank 1 adopt the following decision rule.

Definition 7. The double best response of agent i to an opponents' action profile a_{-i} is the action

$$BR_i^2(a_{-i}) = \arg\max_{x \in A_i} u_i(x, BR_{-i}(x, a_{-i})), \tag{7}$$

where $BR_{-i}(x, a_{-i}) = (BR_1(x, a_{-i}), \ldots, BR_{i-1}(x, a_{-i}), BR_{i+1}(x, a_{-i}), \ldots, BR_n(x, a_{-i}))$ forms the vector of the simultaneous best responses of other agents to the action x chosen by agent i.

As far as the author knows, the decision rule (7) *per se* has not been employed in collective behavior algorithms. In [4] agents choose their actions as a linear combination of the current action and best response. Such approach guarantees collective behavior stability, but appears applicable only if actions are expressed via continuous quantities.

Revert to Example 1 and imagine that all nodes choose the double best response rule. Then for any sequence of moves and any initial power vector the game ends in the optimal equilibrium shown in Fig. 1d (even if the initial powers equal zero). This can be verified directly by applying the rule (7).

There exist networks, where the double best response rule in the original statement leads to "cycling" without connected network attainment. For instance, take the network in Example 2. Nodes c and d located near each other are ready to establish "costly" connection (c, d) and "hope"

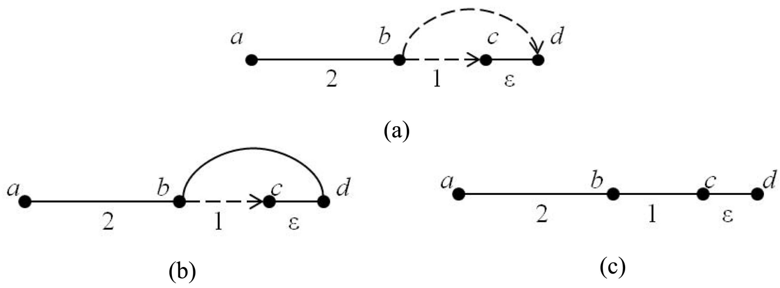


Fig. 2. Equilibria in the network formation game for three nodes: (a) trivial inadmissible equilibrium; (b) another inadmissible equilibrium; (c) suboptimal equilibrium; (d) optimal equilibrium.

that the neighbor will establish "uncostly" connection with node b. The double best response algorithm gets stabilized and yields the network illustrated by Fig. 2a.

If after algorithm termination all nodes switch to simple best response, we obtain one of the networks on Figs. 2b and 2c. Here double best response allows eliminating the most "costly" connections (a, c) and (a, d). The difference between the equilibria on Figs. 2b and 2c is not appreciable.

That double best response surely converges seems unobvious. However, this is the case for all examples experimentally studied in Section 6. It is also possible to provide a formal characterization of node configurations (similar to Fig. 2), where double best response brings to a disconnected network.

4.3. Computational Complexity Restriction

For an agent, evaluation of the simple best response (6) does not require the knowledge of the utilities of other agents; it suffices to know the opponents' action profile a_{-i}. At the same time, double best response calculation implies the knowledge of the utility functions of the opponents and the ability to compute their best responses. This task can be difficult in systems with very many agents. A natural approach lies in restricting the set of agents whose actions can be predicted by agent i. Let us suggest the restricted double best response rule.

Definition 8. The reflexive set R_i of agent i is a set of agents, for which agent i can evaluate the best response (6).

Definition 9. The restricted double best response of agent i to an opponents' action profile a_{-i} is the action

$$BR^2_{i,R_i}(a_{-i}) = \arg\max_{x \in A_i} u_i(x, a_{N \setminus R_i}, BR_{R_i}(x, a_{-i})), \tag{8}$$

where $a_{N \setminus R_i}$ denote the actions of agents beyond the set R_i; $BR_{R_i}(x, a_{-i})$ are the best response of agents belonging to the set R_i.

As a matter of fact, the choice of reflexive sets determines the properties of a collective behavior algorithm. In the limit cases, we have standard best response (under $R_i = \emptyset$) or double best response without restrictions (under $R_i = N$).

In this paper, the reflexive set of node i includes all nodes potentially lying in the range of transmitter i. They satisfy the successful transmission condition (1), if node i adjusts the maximum power of its transmitter. In other words,

$$R_i = \{j \mid j \in N_i^{out}(p_i^{max})\}. \tag{9}$$

By analogy, one can choose $N_i^{out}(p_i^{max}/2)$, $N_i^{out}(2p_i^{max})$, k neighbor nodes, etc.

Furthermore, for each agent it is possible to restrict double best response usage. Suppose that each agent may apply the decision rule (7) only q times; subsequently, it has to switch to the simple best response (6). In this case, we can (a) reduce the computational complexity of the algorithm and (b) avoid cycling, as in Fig. 2.

Both complexity restriction mechanisms make the base of the network formation algorithms proposed in the forthcoming section.

5. NETWORK FORMATION

5.1. Dynamics of Best Responses

Consider an algorithm yielding a connected network in a Nash equilibrium. This algorithm models the behavior of "short-sighted" agents with reflexion rank 0 who employ the best response (6). It was studied in [15], an analogous algorithm was also used in [11]. The algorithm under consideration reproduces the dynamics of iterated best responses (IBRs). Nodes have an arbitrary sequence of moves, e.g., in the increasing order of their mac addresses.

Algorithm 1 (iterated best responses).

1. (*Initialization*). Each node adjusts the maximum power of its transmitter $p_i^0 = p_i^{max}$. Formation of the graph $g^0 = g(p^{max})$ takes place.

2. (*Adaptation*). Successive node i varies its power by the best response rule (6)

$$p_i^{t+1} = BR_i(p_{-i}^t).$$

3. (*Network update*). The new graph

$$g^{t+1} = g(p_i^{t+1}, p_{-i}^t)$$

gets formed.

4. (*Stop*). Steps 2 and 3 are repeated while, at least, one node still varies its power.

The properties of this algorithm are well-analyzed. It was demonstrated in [15] that at each step the algorithm preserves network connectivity. The resulting network turns out connected, either. Moreover, the resulting network represents a Nash equilibrium. Convergence to an equilibrium follows from the fact that the topology control game possesses the order potential function, see [19].

A complete cycle when each node varies its power exactly once (i.e., step 2 repeats n times) is called an iteration of the algorithm. Best response dynamics converges to a Nash equilibrium exactly within one iteration. Such rapid convergence has the following explanation. The best response of each node is reduced to choosing the minimum value of power which still preserves network connectivity. After that, a node will not increase its power and appears unable to decrease it. The stated approach may bring to nonuniform distribution of powers.

As we have illustrated by Fig. 1, the efficiency of the resulting equilibrium depends on the sequence of nodes' actions. Consider a network of 20 nodes generated by the iterated best response algorithm, see Fig. 3a. Here blue solid lines correspond to bidirectional edges, whereas orange dashed lines show monodirectional "superfluous" edges.

5.2. Dynamics of Double Best Responses

This algorithm is constructed by direct analogy with best response dynamics, but agents choose actions by the double best response rule (7).

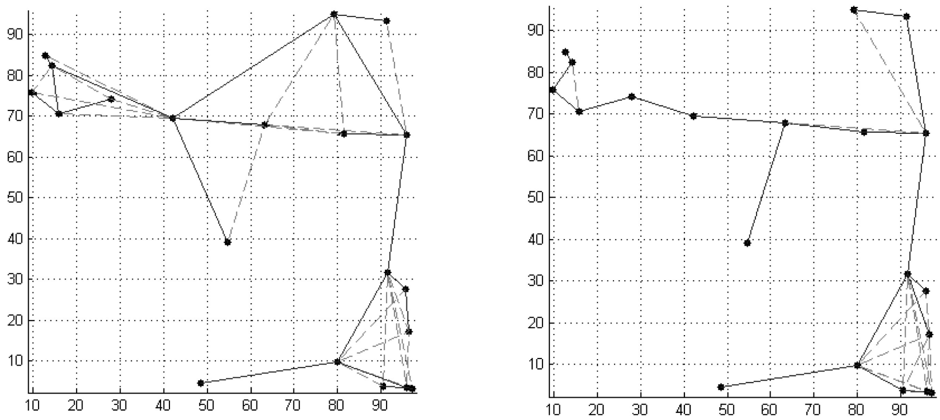


Fig. 3. A network of 20 nodes generated by the algorithms: (a) iterated best response and (b) iterated double best response.

Algorithm 2 (iterated double best responses).

1. (*Initialization*). Each node adjusts an initial power $p_i^0 = p^0$ of its transmitter.

Formation of the graph $g^0 = g(p^0)$ takes place.

2. (*Adaptation*). Successive node i varies its power by the double best response rule (7) or (8),

$$p_i^{t+1} = BR_i^2(p_{-i}^t)$$

or

$$p_i^{t+1} = BR_{i,R_i}^2(p_{-i}^t),$$

where R_i is the reflexive set.

3. (*Network update*). The new graph

$$g^{t+1} = g(p_i^{t+1}, p_{-i}^t)$$

gets formed.

4. (*Stop*). Steps 2 and 3 are repeated while, at least, one node still varies its power.

5. (*Completion*). If the graph g^t is disconnected, all nodes switch to the best response rule (6). Transition to step 2.

At step 2, the rule (7) can be replaced by the rule with restricted reflexive set of agents, see formula (8). The experiments below involve both modifications of the algorithm.

A connected network may not appear after algorithm stop at Step 4. However, one iteration of best response dynamics surely yields a connected network. During execution of Steps 2 and 3, one-way communications possibly occur; their transformation into two-way communications takes place at step 5. Figure 3b demonstrates a network generated by algorithm 2.

5.3. Dynamics with Variable Rank of Reflexion

Double best response has the advantage that a node takes into account the actions of its neighbors. For instance, if nonbeneficial equilibrium variation calls for breaking the network, a node expects that it will be restored by other nodes. In this subsection, we suggest an algorithm combining the flexibility of double best response and the reliability of simple best response.

If a node can improve its utility by simple best response, this node does not employ another rule. In the case when simple best response leads to no utility increase, double best response is used. Each node has a counter c_i, which shows the number of available times to apply double best response. After each time, the counter decrements by 1. If it reaches 0, a corresponding node may utilize simple best response only.

Algorithm 3 (with variable rank of reflexion).

1. (*Initialization*). Each node adjusts the initial power of its transmitter $p_i = p^0$ and the counter $c_i = c^0$.

Formation of the graph $g^0 = g(p^0)$ takes place.

2. (*Adaptation*). Node i evaluates its best response (6), $p_i^{br} = BR_i(p_{-i}^t)$.

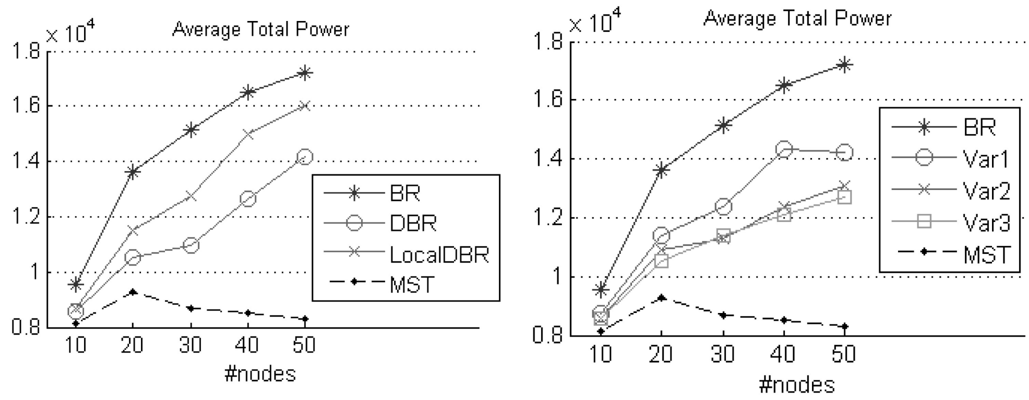
If $u_i(p_i^{br}, p_{-i}) > u_i(p_i, p_{-i})$ and $c_i > 0$, then $p_i^{t+1} = p_i^{br}$.

Otherwise, $p_i^{t+1} = BR_i^2(p_{-i}^t)$ and $c_i = c_i - 1$.

3. (*Network update*). The new graph $g^{t+1} = g(p_i^{t+1}, p_{-i}^t)$ gets formed.

4. (*Stop*). Steps 2 and 3 are repeated while, at least, one node still varies its power.

This algorithm always converges to a connected network at a finite number of iterations. In what follows, we conduct a series of numerical experiments to study the dependence of algorithm efficiency on the initial value of the counter c^0.



(a) (b)

Fig. 4. Efficiency comparison of the algorithms. Axis x answers for the number of network nodes. Axis y corresponds to the total power of all nodes in networks generated by the algorithms: (a) with fixed reflexion rank, (b) with variable reflexion rank.

6. SIMULATION RESULTS

Our experiments have been performed in computing environment *MATLAB*. Nodes have been randomly placed in a square box of dimensions 100×100 "conditional meters" according to the uniform distribution. The placement density has been equal to 10, 20, 30, 40 and 50 nodes per 1 m^2. For each value of the density, we have generated 100 placement configurations. The maximum power p^{max} of nodes has been chosen so that the radius of action makes the half side of the square box.

Figure 4 demonstrates the total power of all nodes in networks yielded by different algorithms depending on the placement density of nodes. Comparison takes place with algorithm 1 involving standard best response (denoted by BR) and with the centralized algorithm constructing the minimum spanning tree (indicated by MST). The paper [6] proved experimentally that the minimum spanning tree approximates the optimal solution with the accuracy of 14–16%.

Note that none of the game-theoretic algorithms has excelled the centralized algorithm of the minimum spanning tree. This is easily explainable, since the utility functions (4) operate only local information, whereas the MST algorithm uses global information on a network.

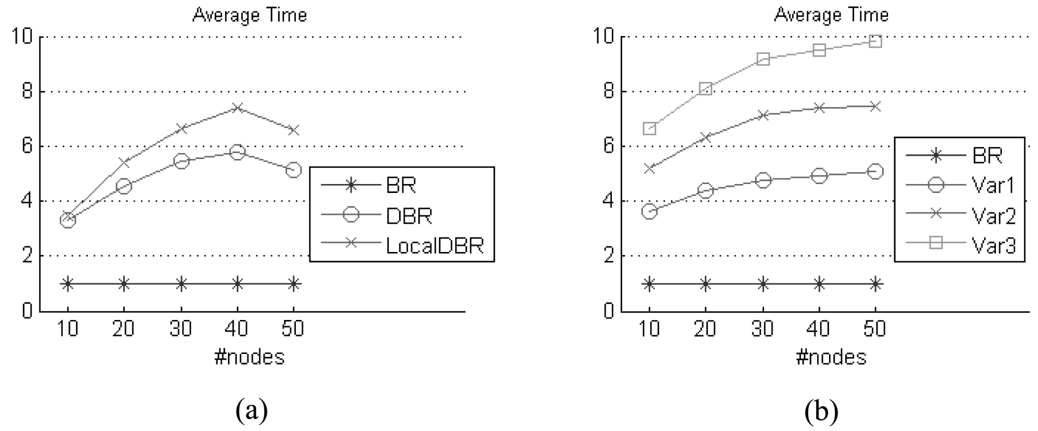


Fig. 5. Speed comparison of the algorithms. Axis x answers for the number of network nodes. Axis y corresponds to the average convergence period of the algorithms: (a) with fixed reflexion rank, (b) with variable reflexion rank.

Figure 4a compares two modifications of algorithm 2 using the double best response (7) (designated by DBR) and the restricted double best response (8), where reflexive sets have been restricted by the maximum radius of action (9) of a node (indicated by *LocalDBR*). Curves DBR and *LocalDBR* illustrate that restricting the reflexive sets of nodes reduces the efficiency of an algorithm. This also increases the convergence period, see Fig. 5a. Therefore, we make the following conclusion. Expanding the reflexive capabilities of an agent improves the efficiency of solution and accelerates network formation.

For algorithm 3 with variable reflexion rank, curves *Var1*, *Var2* and *Var3* show the corresponding results under the reflexion usage limits of 1, 2 and 3, respectively. As we increase the limit, the efficiency of the algorithm grows, but an appreciable difference is observed between the limits of 2 and 3. Similar experiments have been conducted for higher values of the limit; yet, the efficiency remains almost the same, and we omit them on the figures.

Consider the curves in Fig. 4b; clearly, for limits exceeding 1, the algorithm with variable reflexion rank tops its counterpart with fixed double best response in the sense of efficiency for networks with high placement density (40 and 50 on the curves). The convergence period is illustrated by Fig. 5b. Increasing the usage limit of double best response decelerates convergence by 2 iterations on the average.

The experiments have testified that substitution of best response by the double best response rule improves the efficiency of the algorithms and allows constructing networks with smaller total power cost. However, this raises the convergence period of the algorithms. Standard best response for any-size network converges within one iteration. For instance, Fig. 5a demonstrates that double best response for a network of 30 nodes converges during 5.5 iterations on the average. Furthermore, the solution is improved by 30%.

Double best response guarantees the maximum growth of efficiency for networks of medium placement density (20 and 30 nodes per domain). In such networks, the number of admissible equilibria becomes sufficiently large, and achieving same efficiency via several iterations of simple best response is impossible.

Algorithm 3 with variable reflexion rank excels algorithm 2, where nodes employ double best response only. In addition, for appreciable increase in efficiency, it suffices that each node may apply double best response more than once.

7. CONCLUSION

This paper has examined collective behavior algorithms based on the double best response rule which models the behavior of agents with reflexion rank 1. Such algorithms find application in decentralized control of multi-agent technical and socioeconomic systems.

The author has studied usage of double best response dynamics in the topology formation problem of a wireless network. In addition, restriction methods for the computational complexity of reflexion have been analyzed. The author has introduced the notion of the reflexive set of agents for which a given agent can evaluate the best response.

Next, two network formation algorithms have been suggested. According to the first algorithm, nodes adopt double best response only. The second algorithm implies that agents vary their reflexion ranks in the dynamic mode by switching between standard best response and double best response. The efficiency of both algorithms has been compared through a series of numerical experiments. The algorithms involving double best response generate more efficient networks than the algorithm based on simple best response.

The algorithm, where the reflexive set of nodes is restricted by the radius of action of a node transmitter, is inferior to the algorithm, where the reflexive capabilities of nodes are unlimited (both in the aspects of efficiency and convergence speed). The algorithm with variable reflexion rank demonstrates a somewhat better result than the algorithm with fixed restricted double best response.

One may conclude that double best response usage improves the efficiency of collective behavior algorithms. Future research aims at analytical treatment of double best response dynamics. Ideally, it is necessary to establish rigorously the convergence and efficiency in comparison with standard best response dynamics. Furthermore, it is necessary to formulate the specific features of the topology formation game, which are connected with these properties of double best response dynamics. Another direction of promising investigations concerns network formation games with other utility functions or other network formation mechanisms.

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